PREDICTIVE COORDINATION IN TWO LEVEL HIERARCHICAL SYSTEMS

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Abstract: - Noniterative coordination strategy is extended to the case of coordination with prediction. It is applied for optimal resource allocation in a two level system. Appropriate model for steady state optimization is derived.

Key-Words: - Noniterative coordination, multilevel systems, on-line operation, optimization, complex systems.

1 Introduction

The development of the multilevel theory has been strongly influenced by the experience of the human beings in coordinating their efforts reaching an overall goal which can not be achieved individually. The control of hierarchical systems is described by the solution of an optimization problem, decomposed to lower order subproblems, solved by the subsystems and coordinated appropriately. Hence instead of direct solution of a high order optimization problem, the multilevel theory manages and coordinates lower order subproblems [3]. Two main coordination strategies have been worked out: goal and predictive. The goal coordination influences the local goals of the subproblems. The predictive coordination applies a ‘preposition - correction’ protocol. The local subsystems solve and send to the coordinator their prepositions \( x(0) \) found with lack of the global constraints. Using \( x(0) \) the coordinator modifies them towards the global optimal solution \( x^{opt} \) and transmits it to the subsystems for implementation. The noniterative coordination founds on the explicit analytical description and approximation of the dual Lagrange problem.

2 Noniterative Coordination

The coordination in a two level system consists of iterative data transfer between the levels, Fig.1. The coordinator influences with the parameters \( \lambda \) by defining, the optimization subproblems \( Z_i(\lambda), i=1,n \).

**Fig.1. Two level hierarchical system**

The solutions \( x_i(\lambda), i=1,n \) are sent to the coordinator. The last improves \( \lambda \) to \( \lambda^* = \lambda^*(x_i(\lambda)), i=1,n \). Next \( \lambda^* \) is returned to the subsystem for implementation. Thus an iterative communication sequence is performed till finding the optimal coordination \( \lambda^{opt} \), which results in optimal solutions \( x_i^{opt}(\lambda^{opt}), i=1,n \). The iterative coordination yields delays in the control process. They have been reduced by the noniterative coordination, to an information transfer between the system levels in the steps: the coordinator sends to the subsystem initial \( \lambda_0 \); using \( \lambda_0 \) the subsystems solve their problems and evaluate the prepositions \( x(\lambda_0) \); the coordinator corrects \( x(\lambda_0) \) to the global optimal \( x^{opt} \) or evaluate the optimal coordination \( \lambda^{opt} \) without iterative computations; the subsystems evaluate/implement \( x(\lambda^{opt}) \).

This noniterative concept is applied for resource allocation between the subsystems as follows

1. By \( y \) is denoted the common resource, which the coordinator must allocate per subsystems. The subsystems assuming lack of resources, \( y=0 \), evaluate its control \( x(0)=x(y=0) \), which are sent to the coordinator.
2. Assessing the solutions \( x(0) \), the coordinator evaluates the optimal resource allocation per subsystems \( y_i^{opt}=(y_1^{opt}, y_2^{opt}, ..., y_n^{opt}) \). \( n \) - number of subsystem.
3. Using the given resource \( y_i^{opt} \), the subsystem \( i \) evaluates its optimal management \( x_i^{opt} = x_i(y_i^{opt}) \).
3 Noniterative Resource Allocation Model

A global optimization problem, solved by the hierarchical system is stated as

\[
\min \quad F(x) = \sum_{i=1}^{N} f_i(x_i)
\]

\[
g_i(x_i) - y_i = 0, \quad i = 1, \ldots, N
\]

(2) \( y_1 + y_2 + \ldots + y_N = T \)

\[
x_i = (x_{i1}, x_{i2}, \ldots, x_{in}), \quad y_i = (y_{i1}, \ldots, y_{in})
\]

\[
g_i = (g_{i1}, g_{i2}, \ldots, g_{in}); \quad T = (T_1, \ldots, T_m)
\]

The \( y_i \) are resources, allocated to subsystem \( i \) and the total amount of resources is limited to \( T \). If an allocation \( y^*=(y^*_1, \ldots, y^*_n) \) is performed, the global problem is decomposed to \( N \) optimization subproblems,

\[
\min \quad f_i(x_i)
\]

(3)

\[
g_i(x_i) - y_i = 0.
\]

The solution \( x_i(y) \) of (3) is influenced by the allocated resources \( y_i \). The coordination task is to find the optimal allocation \( y^* \), which also is the optimal solution of the global problem (1). Applying the noniterative coordination, the sequence is:

- the subsystem assumes lack of resources, \( y_i = 0, \quad i = 1, \ldots, N \), and solves (3) and found \( x_i(y) \).
- receiving \( x_i(0) \), the coordinator evaluates the optimal allocation

\[
y^* = (y^*_1, \ldots, y^*_N) = y(x_i(0))
\]

(4)

- using \( y_{opt,i} \) \( i = 1, \ldots, N \), the subsystem solves (3) and finds the global solution

\[
x^* = (x^*_1, \ldots, x^*_N)
\]

To be able to perform the noniterative coordination, the relations \( x_i(y), \quad i = 1, \ldots, N \), and \( y = y(x_i(0)) \) must be found in analytical way. It can be linearly approximated.

\[
\frac{dx_i}{dy_i} = \frac{dx_i}{y_i}
\]

(5)

where \( x_i(y) \) is found from (3). The unknown matrix \( \frac{dx_i}{dy_i} \) \( i = 1, \ldots, N \) is found from the Right and Dual Lagrange problem of (3) (the index \( i \) is skipped for simplicity)

\[
\frac{dL(x,y), \lambda(y)}{dx} \bigg| \frac{dL(x,y), \lambda(y)}{dy} \bigg| \frac{\lambda(y)}{0} = 0
\]

(6)

\[
\frac{dL(x,y), \lambda(y)}{dy} = g(x(y)) - y = 0.
\]

The nonlinear system (5) has \((n+m)\) unknown \( (x, \lambda) \) and \((n+m)\) equations. The vectors \( df, dg \) are explicit functions of \( x \). The unknown matrices \( \frac{dx}{dy}, \frac{d\lambda}{dy} \) are found from (A)-(B) by differentiating to \( y \):
\[ f_i(x_i) \approx f_i(x_{i,0}) + \frac{df_i}{dx_i} \bigg|_{x_{i,0}} (x_i - x_{i,0}) + \frac{1}{2} (x_i - x_{i,0})^T \begin{bmatrix} \frac{d^2 f_i(x_{i,0})}{dx_i dx_i} 
 \end{bmatrix} (x_i - x_{i,0}) \]  

(13)

Using (4) the coordination problem is explicitly defined

\[ \min_{\mathbf{y}} \begin{bmatrix} \sum_{i=1}^{N} \frac{1}{2} \mathbf{x}_i^T \mathbf{Q}_i \mathbf{x}_i - \mathbf{x}_i^T \mathbf{b}_i 
 \end{bmatrix} \]

(16)

From [4] the solution of \( y_{i}^{opt} \) and duals \( \pi_i \) is

\[ y_{i}^{opt} = -q_j r_i + a_i \left( \sum_{i=1}^{N} (q_i)_{i-1} \right) + T \]

(17)

Thus if \( f \) is approximated to a quadratic and \( x \) is a linear series at \( x_{i,0} \), where \( x_{i,0} \) solve (3), the optimal resources are found from. Substituting (17) in (4) the suboptimal solution of the problem (1) is found.

4 Numerical Example

The global optimization problem is stated like

\[ \min_{\mathbf{x}} \begin{bmatrix} \frac{1}{2} \left( 3x_1^2 + 2x_2 + 4x_3^2 + x_4 + 5x_5 + 2x_6^2 \right) 
 - 2x_1 - 4x_2 - 3x_3 - x_4 - 6x_5 - 5x_6 
 \end{bmatrix} \]

(18)

Two subproblems are derived

\[ \min f_i(x_i) = \frac{1}{2} x_i^T q_i + x_i^T r_i x_i \]

(19)

\[ G_i(x_i) = a_i x_i = y_i \]

\[ x_i^{(1)} = (x_1, x_2, x_3), \quad x_i^{(2)} = (x_4, x_5, x_6), \]

\[ y_1 = (y_{11}, y_{12}, y_{13}), \quad y_2 = (y_{21}, y_{22}), \quad q_i = \text{diag}(3,1,4), \quad q_2 = \text{diag}(1,5,2), \quad r_i = [-2,-4,-3], \quad r_2 = [-1,-6,-5], \]

\[ a_i = \begin{bmatrix} 2 & 3 & 4 
 1 & 1 & 5 
 -1 & 3 & 4 
 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 5 & 1 & 6 
 2 & 3 & 4 
 \end{bmatrix} \]

The resources \( y_i \) satisfy \( y_1 + y_2 = T = \begin{bmatrix} 8 
 -1 \end{bmatrix} \). Following (4), the optimal subproblems are

\[ x_i^{(1)}(y_i) = x_i^{(1)} + \frac{dx_i^{(1)}}{dy_i} y_i, \quad x_i^{(2)}(y_i) = x_i^{(2)} + \frac{dx_i^{(2)}}{dy_i} y_i \]

\[ x_i^{(1)} = (x_{i1}, x_{i2}, x_{i3}) = \arg \left[ \min f_i(x_i^{(1)}) \right] = \begin{bmatrix} -0.0156 
 0.049 
 0.0041 \end{bmatrix} \]

\[ x_i^{(2)} = (x_{i4}, x_{i5}, x_{i6}) = \arg \left[ \min f_i(x_i^{(2)}) \right] = \begin{bmatrix} 1.767 
 0.6425 
 1.3654 \end{bmatrix} \]

The coordination problem, applying (16), is

\[ \min \begin{bmatrix} \sum_{i=1}^{N} \frac{1}{2} \mathbf{x}_i^T \mathbf{Q}_i \mathbf{x}_i - \mathbf{x}_i^T \mathbf{b}_i 
 \end{bmatrix} \]

(20)

The solution of (20) can be evaluated according to (17)

\[ y_i^{opt} = 8.6719 \quad 1.3022 \quad y_2^{opt} = -0.6719 \quad 2.3022 \]

Applying \( y_i^{opt} \) in (20), the optimal solutions of (18) are

\[ x^{(1)} = \begin{bmatrix} 0.3013 
 2.461 
 0.1714 \end{bmatrix}, \quad x^{(2)} = \begin{bmatrix} x_1^{opt} 
 x_2^{opt} 
 x_3^{opt} \end{bmatrix} = \begin{bmatrix} -1.5512 
 1.0698 
 1.0024 \end{bmatrix} \]

5 Conclusions

The noniterative prediction coordination has been applied for resource allocation in a hierarchical system. The coordination allows the multilevel system to perform the resource allocation without iterative computations, which speeds up the control of the system.

References: